CS171: Cryptography

Lecture 7

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Under the hood

Approach – Stream Ciphers/Block Ciphers

- Heuristic
  - no lower level assumptions
- Formal Definitions Help
- Clear Design Principles

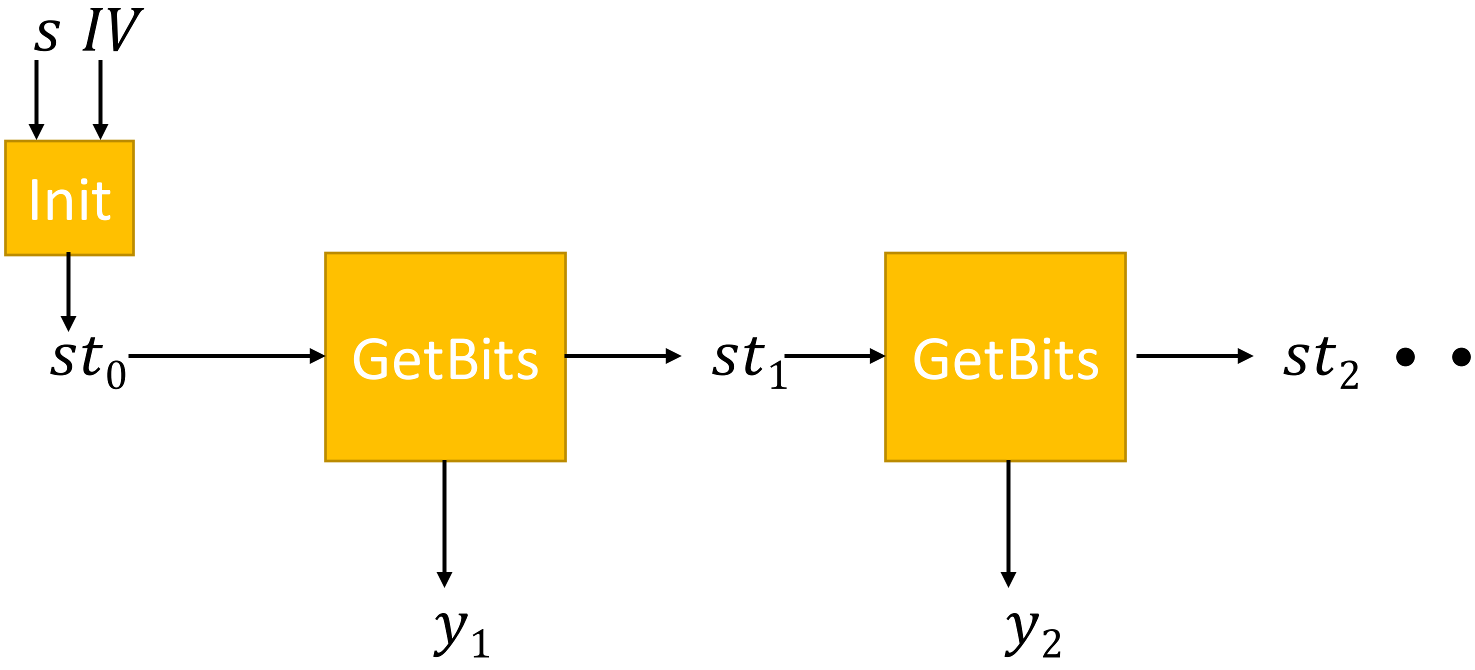
Stream Ciphers

Stream Ciphers

- Init algorithm
  - Input: a key and an *optional* initialization vector (IV)
  - Output: initial state
- GetBits algorithm
  - Input: the current state
  - Output: next bit and updated state
  - Multiple executions allow for generation of desired number of bits

Stream Ciphers

- Use (Init, GetBits) to generate the desired number of output bits from the seed



Security

- Without IV: For a uniform key, output of GetBits should a pseudorandom stream of bits

- With IV: : For a uniform key, and uniform IVs (a*vailable to the attacker*), output of GetBits should be pseudorandom streams of bits (weak PRF)

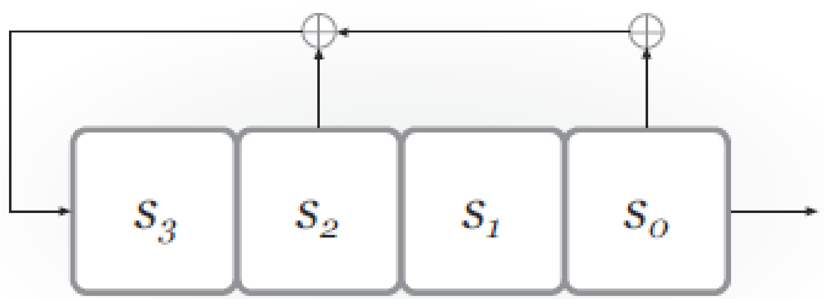
Security

- We care about concrete security and not just asymptotic security

- Efficiency: Keys of length n should give security against adversaries running in time $\approx 2^n$.

LFSRs (Linear Feedback Shift Register)

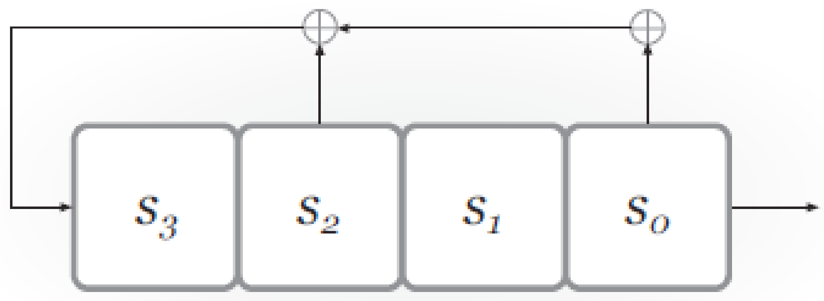
- Degree-n LFSR has n registers
- $s_{n-1} \ldots s_0$ are the contents of the registers
- $c_{n-1} \ldots c_0$ are the feedback coefficients



- Registers updated in each clock cycle

$$s'_{n-1} = \sum c_j s_j \ mod\ 2$$
$$s'_i = s_{i+1} \text{ for } i < n-2$$

LFSR



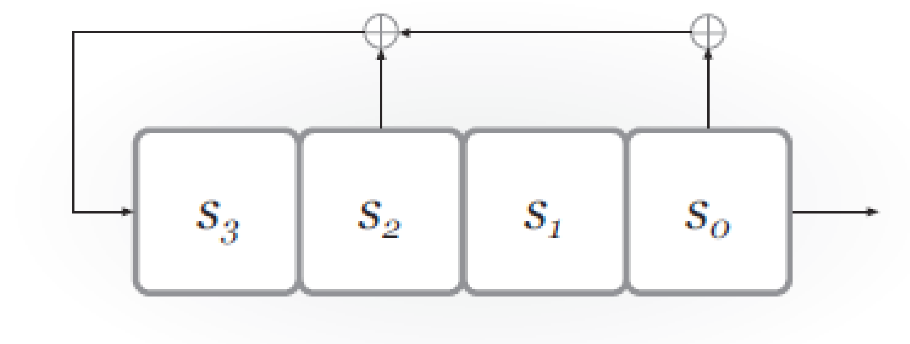
- 0100
- 1010 -> 0
- 0101 -> 0
- 0010 -> 1

Quest for a good LFSR

Output bits will start to repeat for short cycles.

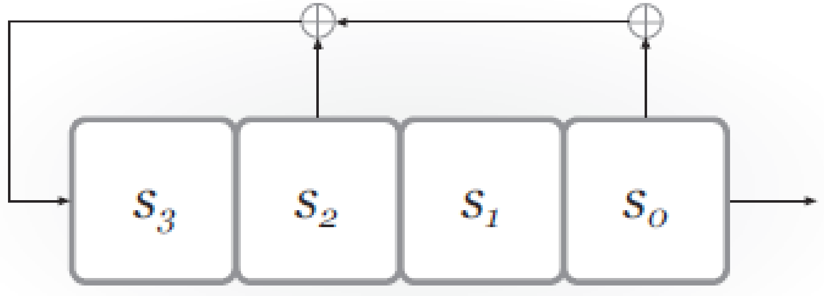
- Intuitively: Should cycle all $2^n - 1$ non-zero states.
- It is known how to set the feedback coefficients to get such an LFSR (also called maximum length LFSR)

- Max length LFSR has good statistical properties but is not cryptographically secure



Can you see an attack?

Attacks on LFSR



- If the feedback coefficients are fixed (and known to the attacker),

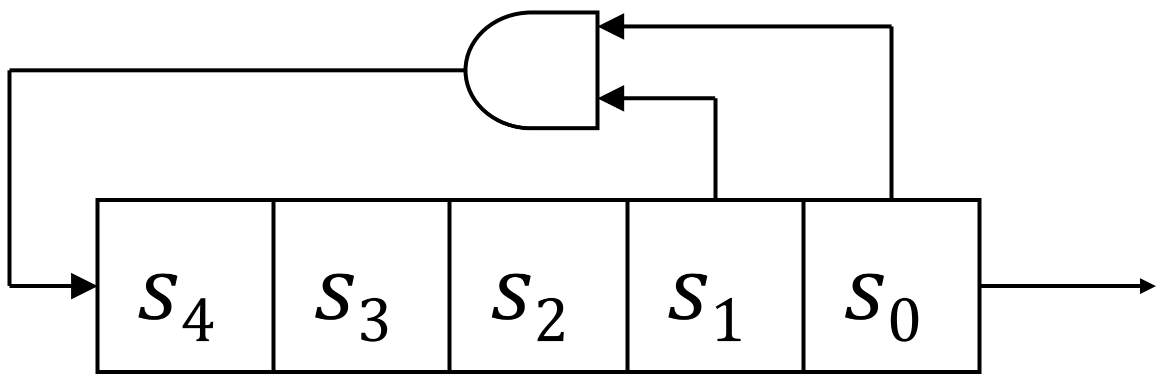
  then the first n output bits fix the key entirely.
- If the feedback coefficients are unknown (and derived from the key),

  then the first $2n$ output bits fix the key and the coefficients. (linear algebra is very powerful)
- Lesson: linearity is *bad* for pseudorandomness

Non-linear FSR

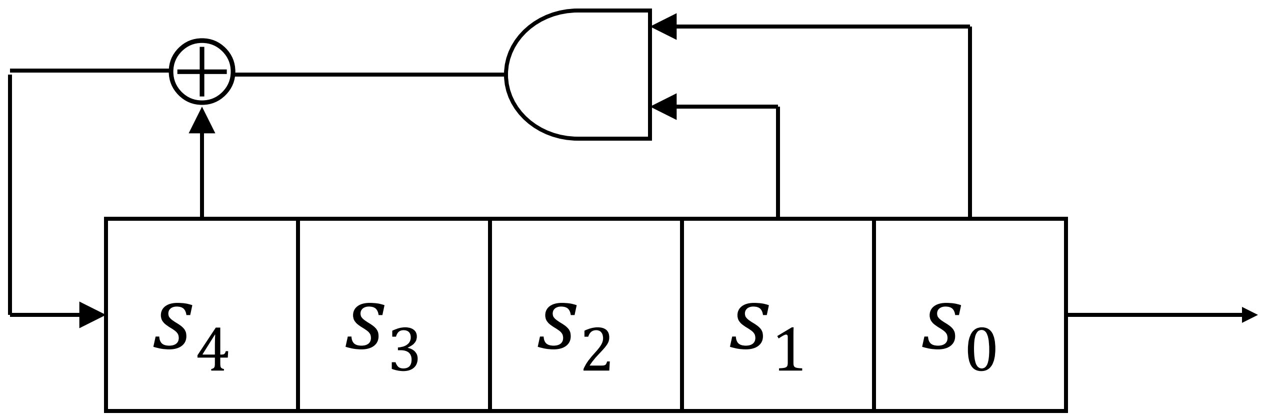
- Adding non-linearity
  - Make the feedback non-linear
  - Make the output non-linear
  - Use multiple LFSRs
  - Mix the above methods.
- Allow for long-cycle and preserve the statistical properties.

Non-linear Feedback



- Is it secure?
- Linear-algebra is not useful!
- However, AND biases the bits!
- How can we fix this?

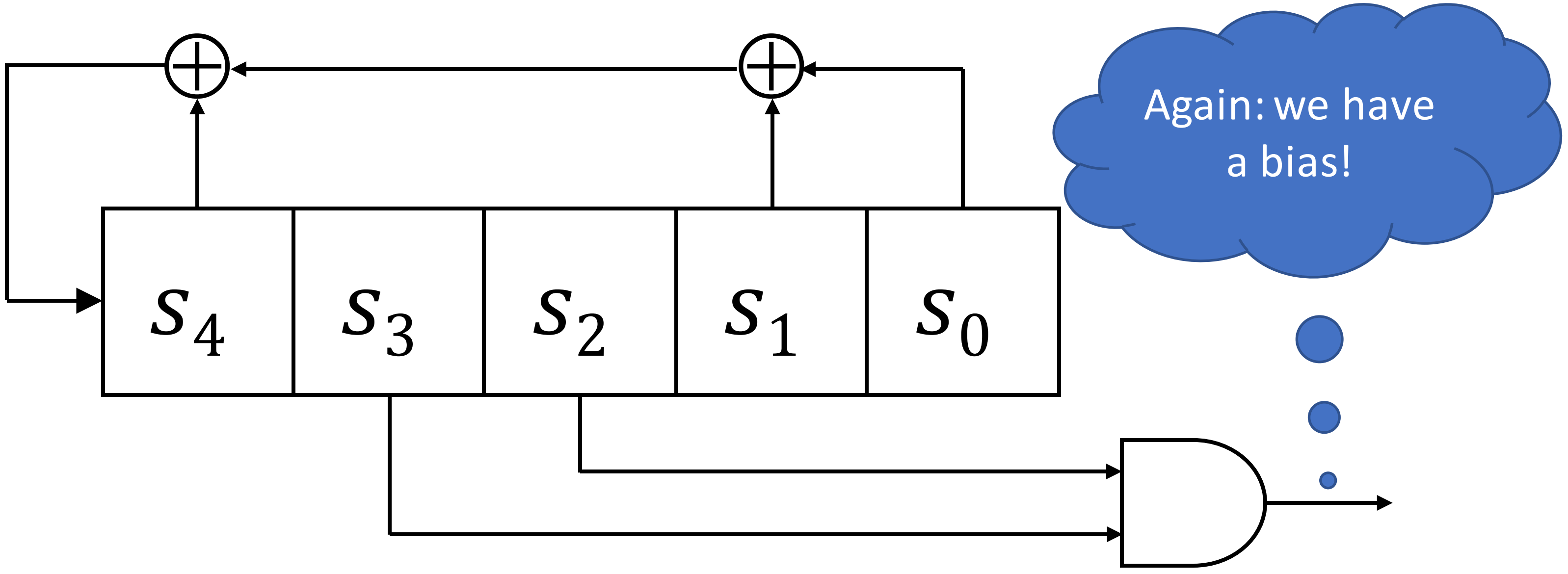
Non-linear Feedback (avoiding bias)



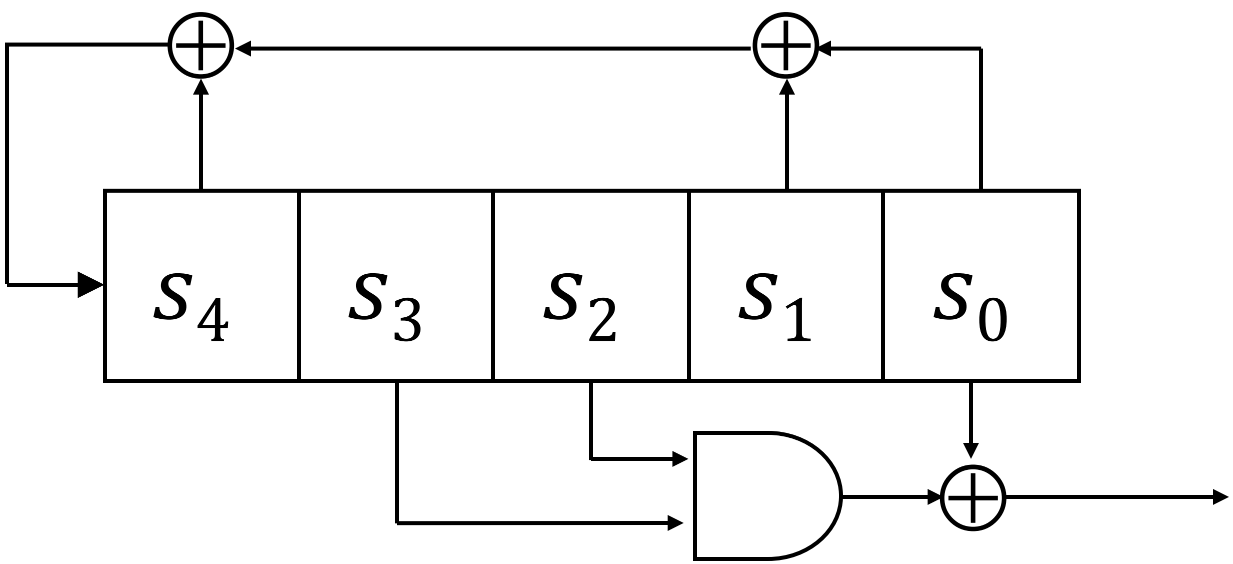
- Use of xor helps remove bias!

Non-linear output

- Update of the LFSR state is linear but the output is obtained as a non-linear function of the state



Non-linear Output (avoiding bias)



Trivium

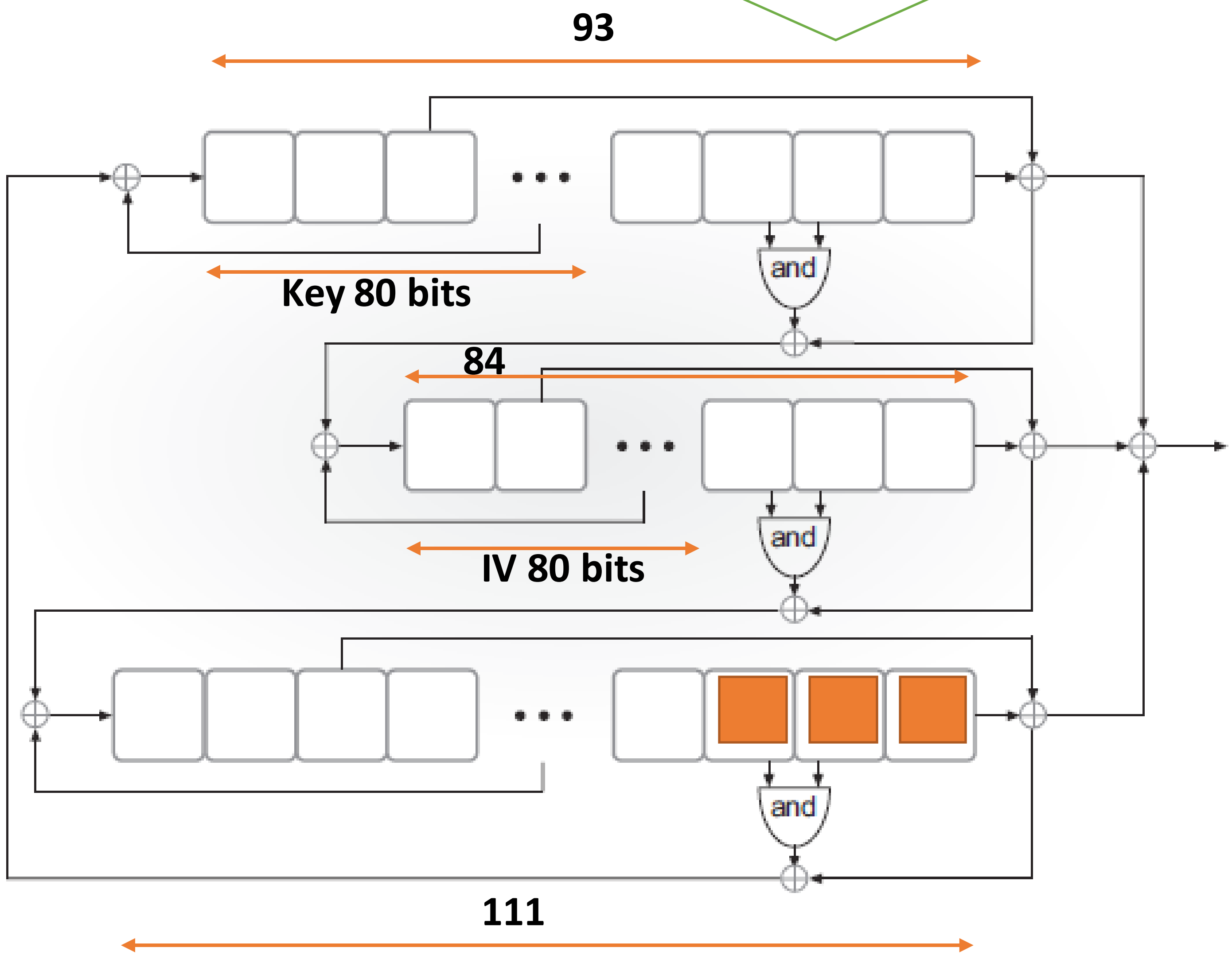
- Designed by De Cannière and Preneel in 2006 as part of eSTREAM competition

- Designed for efficiency in hardware

- No attacks better than brute-force search are known!

Trivium

Set everything else to 0, except the last three registers (of the last FSR) which are set to 1. Then, initialize by executing for $4 \cdot 288$ times and discarding the output bits.



RC4

- Designed in 1987
- Designed for efficiency in software, rather than hardware

- *No longer considered secure*, but still interesting to study
  - Simple description; not LFSR-based
  - Still encountered in practice (WEP 802.11)
  - Interesting attacks

RC4

Set $S[i]$ to be the identity permutation of $\{0 \ldots 255\}$.

One pseudorandom swap and obtain information for a pseudorandom location.

ALGORITHM 6.1
Init algorithm for RC4

Input: 16-byte key k
Output: Initial state (S, i, j)
(Note: All addition is done modulo 256)

```
for i = 0 to 255:
    S[i] := i
    k[i] := k[i mod 16]
j := 0
for i = 0 to 255:
    j := j + S[i] + k[i]
    Swap S[i] and S[j]
i := 0,  j := 0
return (S, i, j)
```

ALGORITHM 6.2
GetBits algorithm for RC4

Input: Current state (S, i, j)
Output: Output byte y; updated state (S, i, j)
(Note: All addition is done modulo 256)

```
i := i + 1
j := j + S[i]
Swap S[i] and S[j]
t := S[i] + S[j]
y := S[t]
return (S, i, j), y
```

Repeat the key to make it 256 byte long.

Each entry of S is swapped with another pseudorandom entry of S.

RC4 used with an initialization vector

- Was not designed for that.
- Set key to be $k = IV||k'$

Attack: Biased 2nd output byte

- Let S_t denote the state of array S after t executions.
- Say S_0 is uniform for simplicity
- Thus, $S_0[2] = 0$ and $S_0[1] = X \neq 2$ happens with probability $\frac{1}{256} \cdot \left(1 - \frac{1}{256}\right) \approx \frac{1}{256}$.

Probability 2nd output byte is 0 is $\approx 1/256 + 1/256$

ALGORITHM 6.2
GetBits algorithm for RC4

Input: Current state (S, i, j)
Output: Output byte y; updated state (S, i, j)
(Note: All addition is done modulo 256)

$i := i + 1$
$j := j + S[i]$
Swap $S[i]$ and $S[j]$
$t := S[i] + S[j]$
$y := S[t]$
return $(S, i, j), y$

After 1 step, $i = 1, j = X$ and $S_1[X] = X$.

After 2 step, $i = 2, j = X$ and $S_2[X] = 0$. $t = X$

$S_1[t] = 0$

More attacks

- Already enough to break EAV-security
- More serious attacks when IV is used
- Attacks can recover keys in WEP

Block Ciphers

Block Ciphers: Recall

- Keyed Permutation
$$F: \{0,1\}^n \times \{0,1\}^\ell \rightarrow \{0,1\}^\ell$$
- n is the key length and ℓ is the block length

- Security: F should be indistinguishable from a uniform permutation over $\{0,1\}^\ell$.
  - Typically, want strong security.
- Interested in concrete security. For key of length n, security is desired against attacker running in time 2^n.

Challenge involved

- F should be indistinguishable from a uniform permutation over $\{0,1\}^{\ell}$.
- If inputs x and x' differ in one bit then what relation between $F_k(x)$ and $F_k(x')$ can we expect?
  - How many bits do we expect to change?
  - Which bits do we expect to change?

Confusion-Diffusion

- Confusion:
  - Small change in input should result in local random change in output

- Diffusion:
  - Local change in output should be propagated to entire output

Design Paradigms

- Substitution-permutation networks (SPNs)
- Feistel networks

Substitution-permutation networks

- Build random-looking permutations on long inputs from random permutations on short inputs

- E.g. Assuming 8-byte block length,

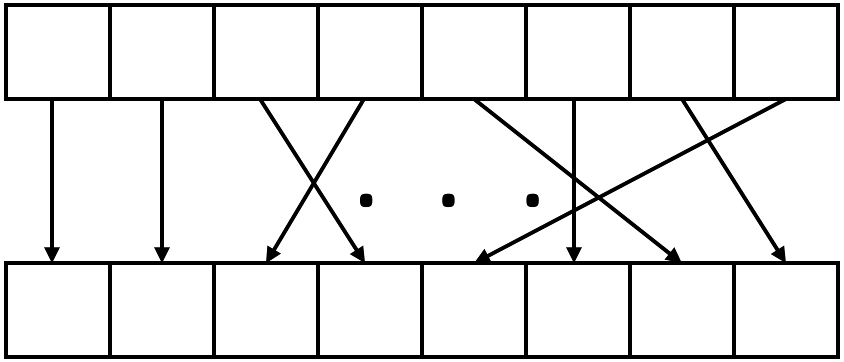
$$F_k(x) = f_{k_1}(x_1) f_{k_2}(x_2) \dots f_{k_8}(x_8)$$

  where each f is a random permutation on $\{0,1\}^8$
  - Is this a PRP?
  - No!
- This has confusion but no diffusion

The key for F is already very big!

Adding Mixing

- $F_k(x) = \ Mix(f_{k_1}(x_1) f_{k_2}(x_2) \dots f_{k_8}(x_8))$ where Mix is a public function.



- This allows for diffusion of the ``propagation of changes”
- So far, given the key, the construction is invertible and hence a permutation

Is this a PRP?

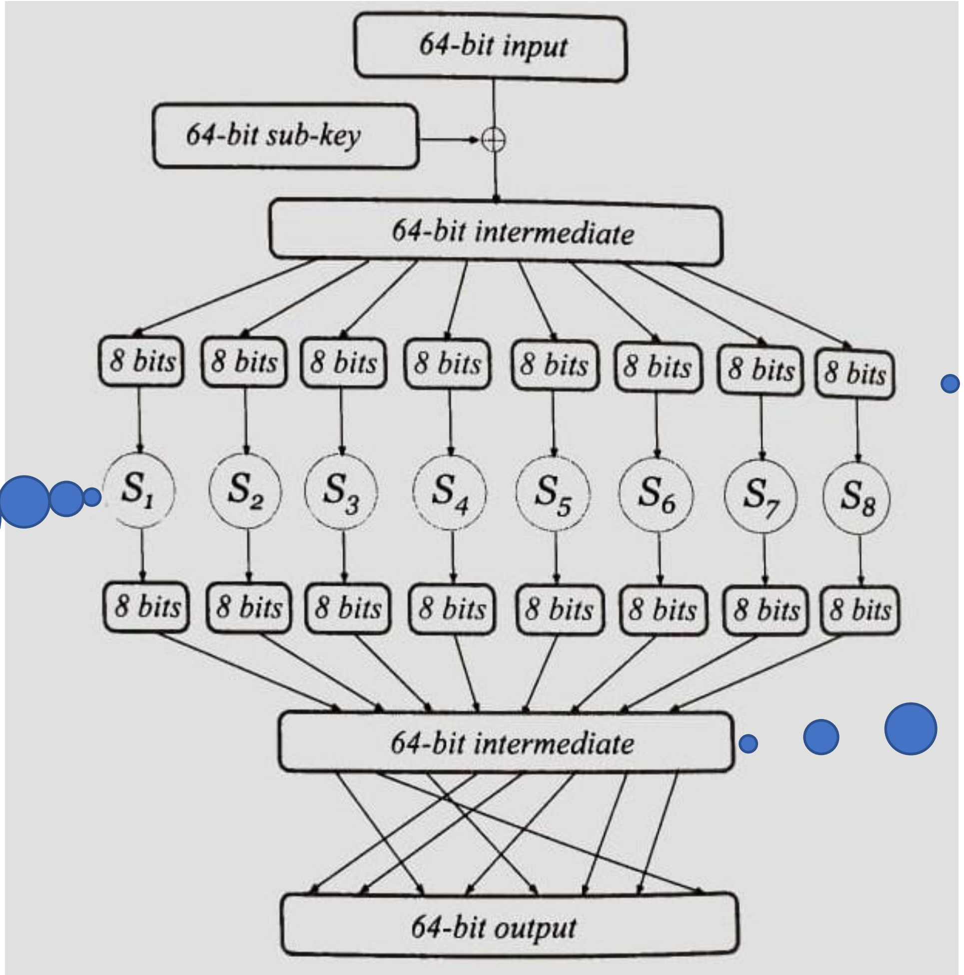
- Not really! Change in input by 1 bit only affects at most 8 output bits.
- What if we repeat the construction (with independent random permutations and a new mixing permutations)?
  - Avalanche effect
  - What is the number of round needed?
    - Carefully decided!
    - Also the mixing permutations need to be carefully chosen!

Making the key smaller

- Using random permutations to start with is not practical.

- Key Mixing: Set $x := x \oplus k$ where k is the key
- Substitution: Set $x := S_1(x_1) \ldots S_8(x_8)$, where x_i is the i-th byte of x.
- Permutation: Permute the bits of x to obtain the output.

Add Mixing Permutation

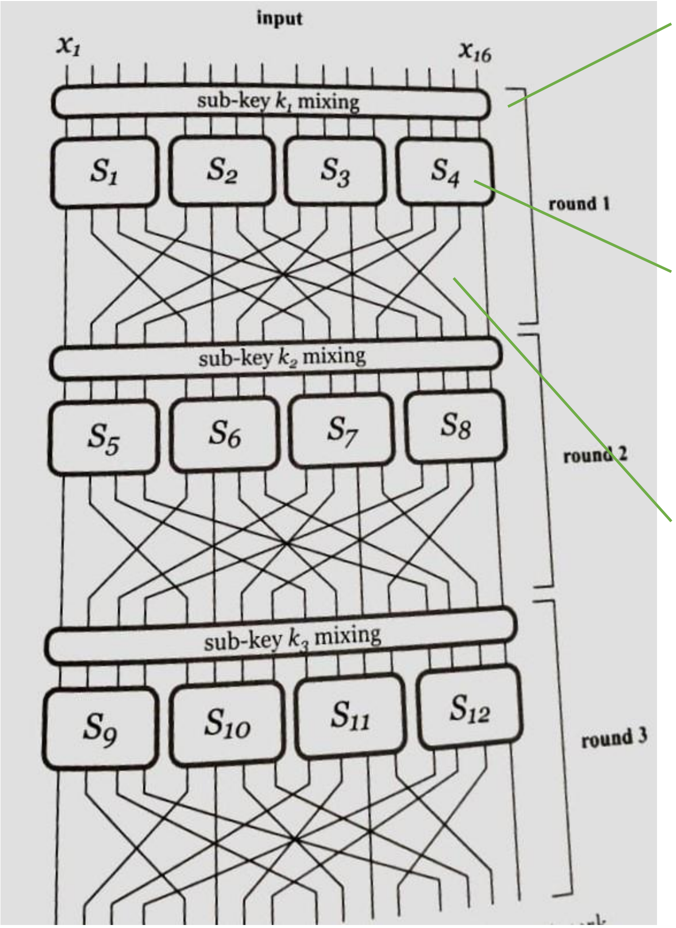


Replace Random permutations with S-boxes!

Invertible!

Public Mixing Permutation!

Repeat with S-boxes



Key Mixing

Passing through S-boxes (Substitution Boxes)

Mixing Permutation

S-boxes and mixing designed with the goal of allowing for an Avalanche effect.

Avalanche effect: Design Principles

- S-boxes and mixing designed simultaneously
  - Small differences should eventually propagate to entire output
- S-boxes: *any* 1-bit change in input causes at least 2-bit change in output
  - Not so easy to ensure!
- Mixing permutation
  - Each bit output from a given S-box should feed into a different S-box in the next round

SPNs

- An r-round SPN has r-rounds of
  - Key-mixing
  - S-boxes
  - Mixing permutation
- One additional key-mixing is done at the last step
- Why?
- Without the final key-mixing the last round is invertible!

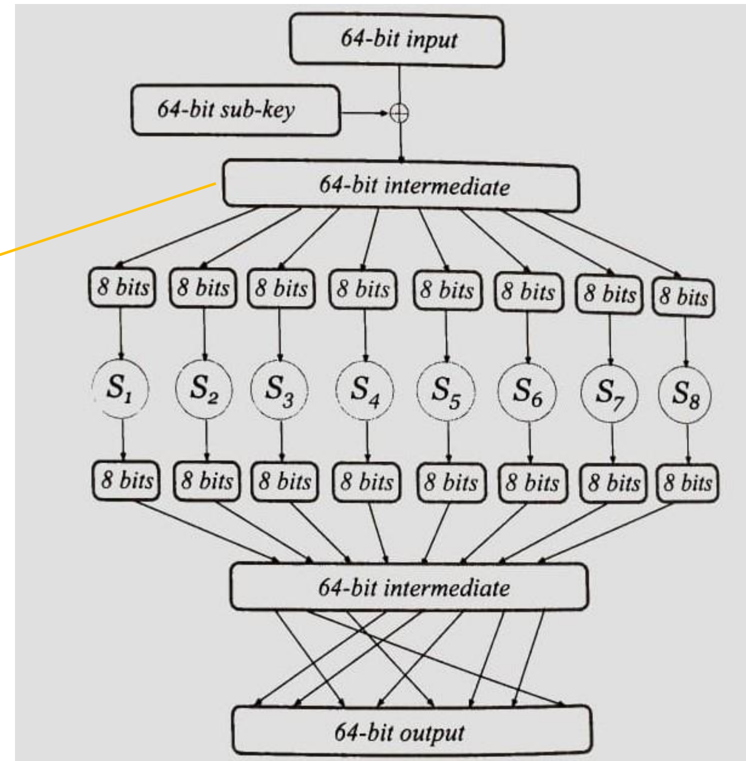
Invertibility and Strong PRP

- Regardless of the number of rounds, it is efficient to invert given the keys.

- Also, S-boxes and mixing permutations are designed such that the avalanche effect applies even when inverting. Thus, we get strong PRPs.

Attacking 1-Round SPN (no output key mixing)

- One Round SPN



Compute z

- Find k given x, y, where $y = F_k(x)$?
- $k = x \oplus z$

Thank You!